Shape Optimization for Stereo Panoramic Digital Vision Systems

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Abstract

There are pixel density limitations to digital cameras, and there is currently much technology push for higher pixel densities. Even if a density is satisfactory for a conventional photograph, it may well be unsatisfactory for a "blow up" of a portion of the image. The situation is compounded for a panoramic image, which pans a full 360 degrees, or for a split mirror system giving a stereo image via the one camera lens. Any system would be at its limits when stereo panoramic real time imaging is used for such applications as mobile robotics with image based range finding, and there is need to "blow up" features for recognition and range finding. Can mathematics help in the design and optimization of such system?

Panoramic images are actually standard photographs or video images of concentric, hemi-spherical mirrors which reflect a hemispherical scene to the camera lens. Thus with such a hemispherical mirror attached by its flat face to the center of the ceiling of a room, and the camera facing the mirror vertically aligned with the mirror axis, then the image is a warped image of the room. De-warping such an image, assumed for the moment to be infinite in resolution, would achieve the same information as a camera scanning horizontally the full 360 degrees to construct a panoramic image.

For a finite resolution camera used in panoramic imaging, the initial warped image has a uniform pixel density, and the mapping of these pixels in the nonlinear de-warping process results in their non-uniform distribution. It is clear that more pixels are assigned to objects near the ceiling

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than otherwise, since these map to the periphery of the warped image, which has more pixels than the central image area. Thus in the de-warped image, objects near the ceiling are less blurred than otherwise. In stereo concentric panoramic images, the peripheral image is higher resolution than the central image. This makes range finding from stereo images quite a different process than for conventional stereo images when the two images have the same resolutions throughout.

A challenge taken up in this research has been to perform panoramic mirror shape optimization to achieve uniform pixel density de-warped images [1, 2]. The particular focus has been to stereo panoramic imaging, as is potentially useful in mobile robotic navigation, surveillance and other applications. Certainly, such vision systems go beyond the human vision system, but are perhaps closer to vision systems of some birds, which achieve almost a total spherical image from two hemispherical eyes.

They key tool for our optimization is a nonlinear differential equation defining a mirror profile to achieve a specified mirror gain profile on the mirror surface. The mirror gain is a sensitivity parameter, giving the ratio of the change in light ray incidence angle to the change in light ray reflected angle. These equations are based on an assumption of an ideal pinhole camera, and Snell's Laws of mirror reflection, which incidentally result from a minimum ray path distance optimization. Earlier research by Srinivasan and Chahl [3], developed the nonlinear equations to achieve a constant gain profile, so facilitating the image dewarping.

In the research reported here, by allowing a specified gain profile in the nonlinear equations defining a mirror profile, the mirror designer is able to design panoramic mirrors with a specified mirror gain profile. This allows, for example, the mapping of a uniform radial pixel density in the warped image to any desired radial pixel density in the de-warped image. This translates to vertical pixel density in the de-warped image. Now, the horizontal pixel density in the de-warped image is constrained by the geometry, to decrease linearly form the top of the image to the bottom. It makes sense then, to seek a uniform solid angle pixel density by having the vertical density increase linearly from the top of the image to compensate for the decreasing horizontal density.

Of course one could argue that this notion of achieving a uniform solid angle density in an image is not going to work well in practice, because higher vertical resolution can not always compensate for a low horizontal resolution. And yes, it is not difficult to construct example images to support this skepticism. But, surprisingly, for real world images, at least the human eye is able to benefit from this particular form of uniform solid angle resolution. There are many examples when features are totally blurred beyond recognition with conventional panoramic images and which can be recognized with the uniform solid angle resolution. For machine vision which uses stereo concentric warped images, de-warped to more conventional images but panoramic, for range finding, again there are examples where feature location in the two images is made precise because of a uniform solid angle resolution.

One advantage of a double coaxial panoramic mirror system for panoramic stereo, is that any image feature is duplicated on the same radial line in the warped image, and there is precise horizontal correspondence in de-warped images. This means that the correspondence issue for range finding in this situation is defused.

The workshop seminar presentation will concentrate on illustrating our innovations in panoramic stereo imaging for range finding. The material will be taken from a number of recent references [4, 1, 5], including a PhD thesis [2]. The first report of a coaxial stereo panoramic mirror system is in [6]. Some related background material is found in references

References

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[4] T. L. Conroy and J. B. Moore, "Reflective surfaces for stereo panoramic vision systems." Submitted revised to *Int. Journal Computer Vision*, April 2000.

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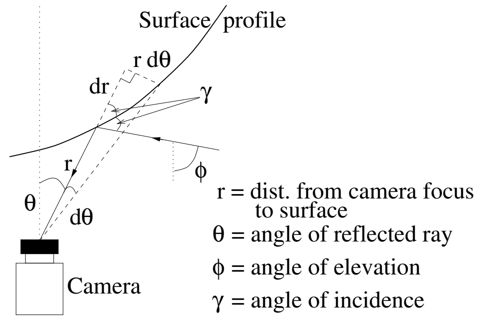
BACKGROUND - The Constant Gain Mirror

- From Chahl and Srinivasan (1997).

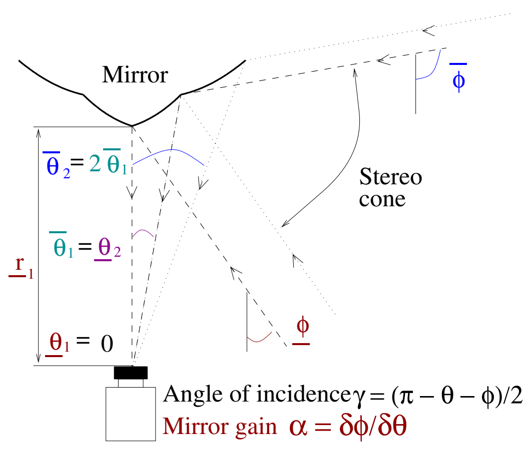


- From above, $\gamma = \tan^{-1}(r\mathrm{d}\theta/\mathrm{d}r)$
- From the law of reflection, $2\gamma + \theta + \phi = \pi$
- Differentiating w.r.t. θ and equating: $\frac{\mathrm{d}}{\mathrm{d}\theta}[\tan^{-1}(r\mathrm{d}\theta/\mathrm{d}r)] = -(1 + \mathrm{d}\phi/\mathrm{d}\theta)/2$
- Integrating and rearranging achieves profile differential equation.

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = r \cot\left[-\frac{1}{2}\int\left(1 + \frac{\mathrm{d}\phi}{\mathrm{d}\theta}\right)\mathrm{d}\theta\right]$$

THE STEREO CONSTANT GAIN MIRROR

- The general stereo constant gain mirror, with $\alpha > 1$:



- Equation of lower profile: $\sin\left[\underline{\gamma_1} + (\underline{\theta_1} - \theta_1)(1+\alpha)/2\right] = (r_1/\underline{r_1})^{-(1+\alpha)/2} \sin \underline{\gamma_1}$
- Equation of upper profile: $\sin\left[\underline{\gamma_2} + (\underline{\theta_2} - \theta_2)(1+\alpha)/2\right] = (r_2/\underline{r_2})^{-(1+\alpha)/2} \sin \underline{\gamma_2}$

EQUATIONS FOR THE VARIABLE GAIN MIRROR

- The mirror gain:

$$\begin{aligned}\alpha(\theta) &= \frac{\delta\phi(\theta)}{\delta\theta} \\ &= B_\alpha[\tan(\theta) + \tan^3(\theta)]\end{aligned}$$

where B_α is a constant:

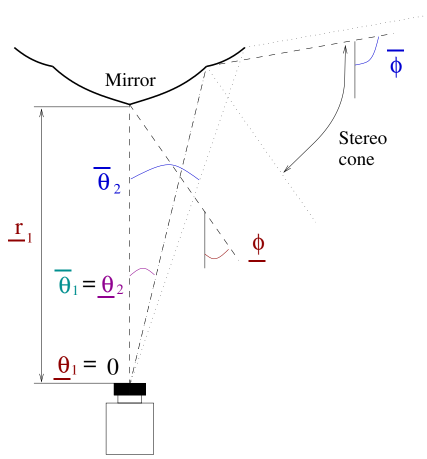
$$B_\alpha = \frac{2(\overline{\phi} - \underline{\phi})}{\tan^2(\overline{\theta}) - \tan^2(\underline{\theta})}$$

- The mirror equation:

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = r\cot\left[-\frac{1}{2}\int(1+\alpha(\theta))\,\mathrm{d}\theta\right]$$

- A differential equation solver is needed to find solutions to the mirror equation over the mirror surface since α is a function of θ.

THE STEREO VARIABLE GAIN MIRROR



- Equation of lower profile: $\frac{\mathrm{d}r_1}{\mathrm{d}\theta_1} = r_1\cot\left[-\frac{1}{2}\int(1+\alpha(\theta_1))\,\mathrm{d}\theta_1\right]$
- Equation of upper profile: $\frac{\mathrm{d}r_2}{\mathrm{d}\theta_2} = r_2\cot\left[-\frac{1}{2}\int(1+\alpha(\theta_2))\,\mathrm{d}\theta_2\right]$
- Mirror discontinuity point: $\overline{\theta_1} = \tan^{-1}\left[\left(\frac{\tan^2(\overline{\theta_2})+\tan^2(\underline{\theta_1})}{2}\right)^{1/2}\right]$